Return-risk comparisons

- Sometimes, it is very easy to predict a choice of an individual between two lotteries (G(.) and F(.)), even if we do not know her exact attitude towards risk
- The first case is when F(.) yields unambiguously higher returns than G(.). In this case (almost) anyone will pick F(.)
- The second case is when G(.) and F(.) give the same returns, but F(.) is less risky. In this case any risk averter will pick F(.).

First-order stochastic dominance

- Def.: The distribution *F*(.) first-order stochastically dominates *G*(.) if, for **every nondecreasing** *u*:

$$\int u(x)dF(x) \geq \int u(x)dG(x)$$

i.e. if every expected utility maximizer prefers *F*(.) to *G*(.)

- Prop.: The distribution *F*(.) first-order stochastically dominates *G*(.) iff *F*(*x*) ≤ *G*(*x*) for every *x*
- *F*(.) is a transformation of *G*(.), such that each payoff realization in *G*() is subject to an „upward probabilistic shift (or spread)".
- 1-st order s.d. implies that *F*(.) has higher mean than *G*(.). The converse does not hold.
- graphs

Example: 1st order s.d.

- Lottery G:

	1	2	3	4	5
d.f.	1/2	0	0	1/2	0
c.d.f	1/2	1/2	1/2	1	1

- Lottery F:

	1	2	3	4	5
d.f.	0	1/4	1/4	0	1/2
c.d.f	0	1/4	1/2	1/2	1

- F(.) 1st order stochastically dominates G(.)

Example: 1st order s.d.

- Lottery G:

	1	2	3	4	5
d.f.	1/2	0	0	1/2	0
c.d.f	1/2	1/2	1/2	1	1

- Lottery F:

	1	2	3	4	5
d.f.	1/6	2/6	1/4	0	1/4
c.d.f	1/6	1/2	3/4	0	1

- F(.) does not dominate G(.)

Second-Order Stochastic Dominance

- Def.: Suppose *G*(.) and *F*(.) have the same mean. The distribution *F*(.) second-order stochastically dominates (is less risky than) *G*(.) if, for **every concave** *u*:

$$\int u(x)dF(x) \geq \int u(x)dG(x)$$

- i.e. if every risk-averter prefers *F*(.) to *G*(.)
- *G*(.) is a mean – preserving spread of *F*(.)
- graphs

Example: 2nd order s.d.

- Lottery F:

	1	2	3	4	
d.f.	0	1/2	1/2	0	
c.d.f	0	1/2	1	1	

- Lottery G:

	1	2	3	4	
d.f.	1/4	1/4	1/4	1/4	
c.d.f	1/4	1/2	3/4	1	

- F(.) 2nd order stochastically dominates G(.)

Investing in risky assets

- If a risk averter is faced with only 2 investment options, one risky and one riskless, she will invest part of her wealth in the risky asset, no matter how risky she is.
- If a risk averter is faced with several investment options, none of which stochastically dominates the other, she will invest part of her wealth in every asset,

Correlated returns

- Suppose that the return on an asset depend on a realization of some random process, i.e. „state of the world"
- The table below show an example of rates of return for 3 assets: Bonds, Stocks and Real Estate

Asset\State	s_1	s_2	s_3
B	5	5	5
S	3	6	7
R	6	7	8

- Notice that R is better than B no matter what the state of the world will be. We say that R dominates B and conclude that B should never be chosen by a rational investor (regardless of risk aversion and the probabilities of the states)

Another example

Asset\State	s_1	s_2	s_3
B	5	5	5
S	3	5	6
R	2	6	8

- In this case none of the assets dominates another. However, notice that you can create a portfolio: put 50% of money in B and 50% in R. Such portfolio dominates S: no matter what happens, your average return will be higher than the return from S. We say that a B-R mix dominates S, and conclude that S should never be chosen by a rational investor